e content for students of patliputra university

B. Sc. (Honrs) Part 1paper 1

Subject:Mathematics

Title/Heading of topic:Set, Subset &Power set

By Dr. Hari kant singh

Associate professor in mathematics

Rrs college mokama patna

Set

- **Definition**: A **set** is a (unordered) collection of objects. These objects are sometimes called **elements** or **members** of the set. (Cantor's naive definition)

- **Examples:**
  - **Vowels in the English alphabet**

    V = { a, e, i, o, u }
  - **First seven prime numbers**.

    X = { 2, 3, 5, 7, 11, 13, 17 }

Representing sets

Representing a set by:

1) Listing (enumerating) the members of the set.

2) Definition by property, using the set builder notation

{x| x has property P}.

Example:

- Even integers between 50 and 63.

1) E = {50, 52, 54, 56, 58, 60, 62}

2) E = {x| 50 <= x < 63, x is an even integer}

If enumeration of the members is hard we often use ellipses.

Example: a set of integers between 1 and 100

- **A= {1,2,3 …, 100}**

Equality

Definition: Two sets are equal if and only if they have the same elements.

Example:

- **{1,2,3} = {3,1,2} = {1,2,1,3,2}**

Note: Duplicates don't contribute anything new to a set, so remove them. The order of the elements in a set doesn't contribute anything new.

Example: Are {1,2,3,4} and {1,2,2,4} equal?

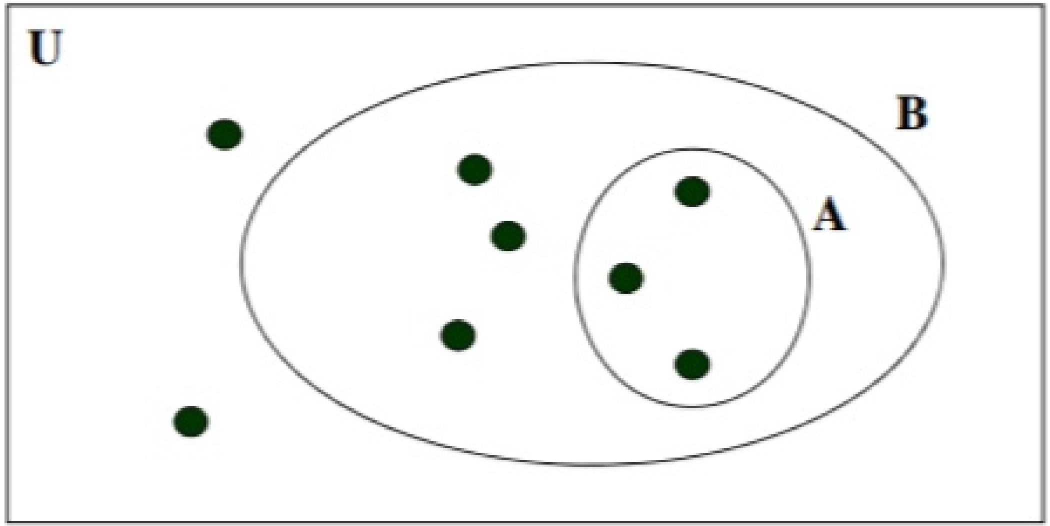
No!

Special sets

- **Special sets:**
  - **The <u>universal set</u>** is denoted by **U:** the set of all objects under the consideration.
  - **The <u>empty set</u>** is denoted as **∅ or { }.**

A Subset

- **Definition**: A **set** A is said to be a **subset** of B **if and only if** every element of A is also an element of B. We use **A ⊆ B** to indicate **A is a subset of B**.



- Alternate way to define A is a subset of B:

  $\forall x\ (x \in A) \rightarrow (x \in B)$

Empty set/Subset properties

Theorem $\varnothing \subseteq S$

- **Empty set is a subset of any set.**

Proof:

- Recall the definition of a subset: all elements of a set A must be also elements of B: $\forall x\ (x \in A \rightarrow x \in B)$.
- We must show the following implication holds for any S
  $\forall x\ (x \in \varnothing \rightarrow x \in S)$
- Since the empty set does not contain any element, $x \in \varnothing$ is **always False**
- Then the implication is **always True.**

End of proof

Subset properties

Theorem: $S \subseteq S$

- **Any set S is a subset of itself**

Proof:

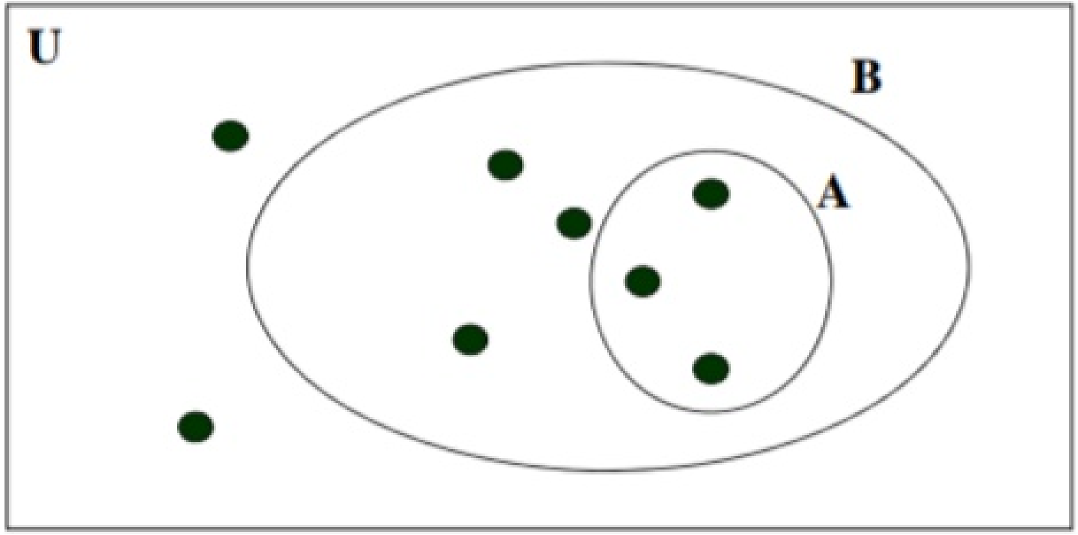
- the definition of a subset says: all elements of a set A must be also elements of B: $\forall x\ (x \in A \rightarrow x \in B)$.
- Applying this to S we get:
- $\forall x\ (x \in S \rightarrow x \in S)$ which is trivially **True**
- End of proof

Note on equivalence:

- Two sets are equal if each is a subset of the other set.

A proper subset

Definition: A **set A** is said to be a **proper subset** of B if and only if **$A \subseteq B$** and **$A \neq B$**. We denote that A is a proper subset of B with the notation $A \subset B$.



Example: A={1,2,3} B ={1,2,3,4,5}

Is: $A \subset B$? Yes.

Power set

Definition: Given a set S, the **power set** of S is the set of all subsets of S. The power set is denoted by **P(S).**

Examples:

- Assume an empty set $\emptyset$
- What is the power set of $\emptyset$? $P(\emptyset) = \{ \emptyset \}$
- What is the cardinality of $P(\emptyset)$? $| P(\emptyset) | = 1$.

- Assume set $\{1\}$
- $P(\{1\}) = \{ \emptyset, \{1\} \}$
- $|P(\{1\})| = 2$

Power set

- $P(\{1\}) = \{\varnothing, \{1\}\}$
- $|P(\{1\})| = 2$

- Assume $\{1,2\}$
- $P(\{1,2\}) = \{\varnothing, \{1\}, \{2\}, \{1,2\}\}$
- $|P(\{1,2\})| =4$

- Assume $\{1,2,3\}$
- $P(\{1,2,3\}) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$
- $|P(\{1,2,3\}| = 8$

- **If S is a set with $|S| = n$ then $|P(S)| = ?$**